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CS 600WS – Advanced Algorithms

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Homework 10

I pledge my honor that I have abided by the Stevens Honor System.

1. R-19.2 Suppose two teams, the Anteaters and the Bears, have a long rivalry in basketball. Suppose further that in any given game, the Anteaters will beat the Bears with probability 2/3, independent of any other games that they play. Give a bound on the probability that, in spite of this, the Bears will win a majority of n games that they play.
   1. Using Chernoff Bounds, , ,
2. C-19.7 Suppose that there is a collection of 3n distinct coupons, n of which are colored red and 2n of which are colored blue. Suppose that each time you go to a ticket window to get a coupon, the clerk ﬁrst randomly decides, with probability 1/2, whether he will give you a red coupon or blue coupon and then he chooses a coupon uniformly at random from among the coupons that are that color. What is the expected number of times that you must visit the ticket window to get all 3n coupons?
   1. First, to make this easier to read, set , and . Therefore, and . The ½ is there because the probability of getting is initially ½. Therefore, , which simply equals since it’s significantly higher than and all the red coupons will be acquired in the trips where acquiring blue tickets is unsuccessful.
3. A-19.2 In the Mega Millions lottery game, a player picks ﬁve ***lucky*** numbers, in the range from 1 to 56, and one additional ***Mega*** number, in the range from 1 to 46. In order to win the jackpot, a player must match all six numbers. If there is no jackpot winner for a given drawing, then the jackpot is rolled into the next drawing. Suppose that every time a lottery ticket is sold it is chosen as an independent random pick of ﬁve lucky numbers and a Mega number. What is the expected number of Mega Millions lottery tickets that must be sold for a given drawing to guarantee with 100% certainty that there is a winner?
   1. To guarantee, with 100% certainty, that the winning ticket was purchased, you could also guarantee that all the unique tickets were purchased, thus simply boiling this down to the Coupon Counting Problem. Therefore, where .